Integer and fractional charge solitons in modulated strips in the fractional quantum Hall regime

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Abstract

We propose the existence and study the solitonic excitations in two kinds of samples in the fractional quantum Hall regime. One is a strip modulated by a one-dimensional array of gates. The other is made of two parallel strips coupled by a one-dimensional array of tunnel barriers. We predict the existence of integer charge solitons in the first case, and fractional charge solitons in the second case. We study the two cases both in the dissipative and in the inertial limits.

1 Introduction

Samples in the fractional quantum Hall (FQH) regime have been studied extensively, since they are in a novel quantum state and exhibit an interesting spectrum of excitations [1]. Here we predict the existence and study the properties of solitonic excitations in such systems. Motivated by the investigations of fluxons in quasi one-dimensional (1D) long Josephson junctions [2] and charge solitons in 1D arrays of normal [3] and Josephson [4] junctions, we consider the two systems shown in Fig. (1).

Both systems are made of strips (quasi 1D) of samples in the FQH regime with imposed geometrical restrictions. This type of devices can be fabricated and studied experimentally [5]. In the first system, an array of gates (narrow bridges) is imposed on the strip. We refer to this as case I. The second system

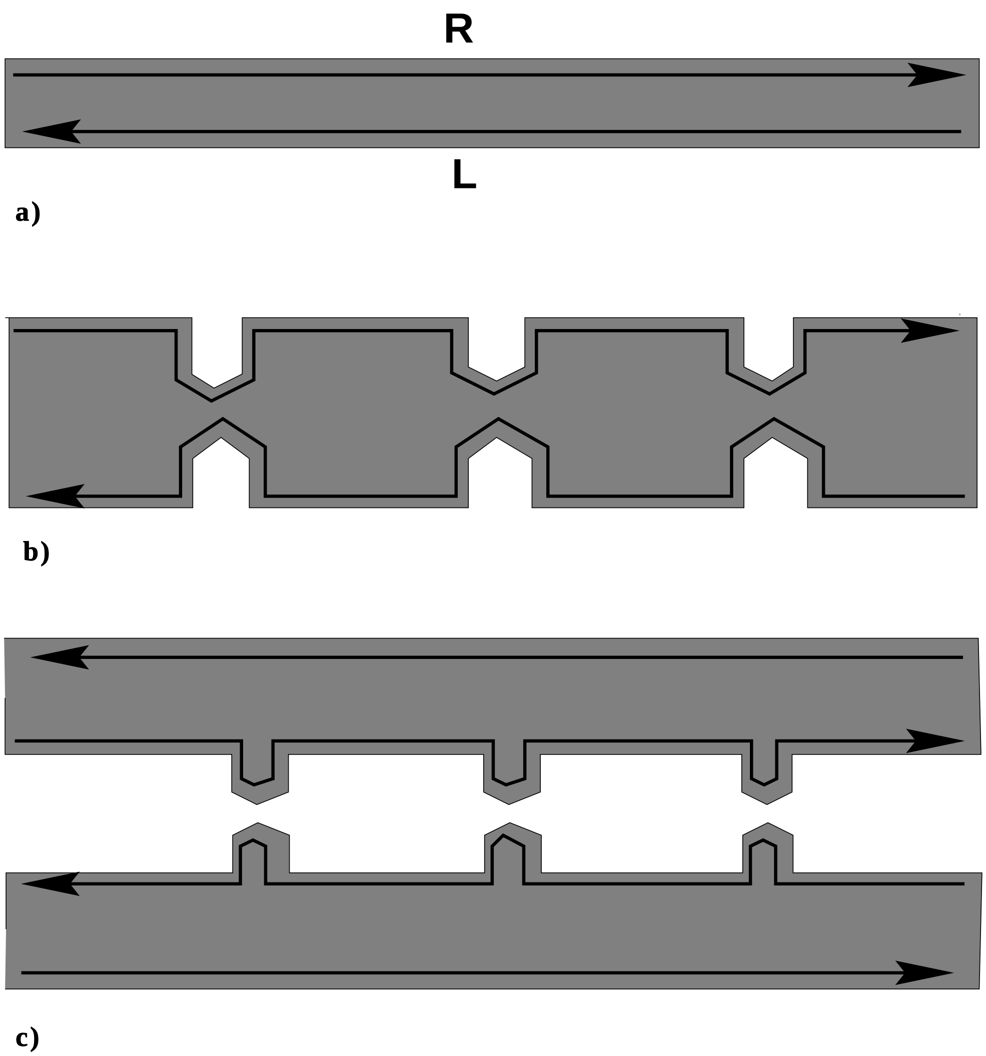

Figure 1: a) The right and left edge currents; b) System I: FQH strip with gates; c) System II: Two FQH strips connected by tunnel barriers.

(case II) is composed of two separate parallel strips connected by an array of tunnel barriers. Both systems can be understood as quasi 1D arrays of FQH islands connected by tunnel barriers. We include in our model the charging energy of the islands in the two cases.

In the absence of gates, a strip in the FQH regime is circulated by edge currents. A gate induces tunneling of quasiparticles between the two edges. The result is charge localization-like (of an integer charge) and Coulomb blockade-like effects. Hence case I is analogous to a 1D array of serially coupled normal [3] or Josephson [4] junctions. In this case we predict the existence of integer charge solitons. We discuss two limits according to the system's parameters: 1. the dissipative limit, in which the solitons behave as overdamped classical particles; 2. the inertial limit, in which the kinetic energy becomes important, and the solitons behave as underdamped semi-classical or quantum particles.

Case II, in which the charging energy of the islands is included, is analogous to a 1D array of Josephson junctions coupled in parallel. A continuous version of this system, i.e., two parallel strips coupled by a continuous thin tunnel barrier (instead of an array of discrete barriers) was considered by Wen [6]. He showed that the system is in the deep quantum limit. We find that this additional charging energy drives the system towards the semi-classical regime, and can produce free solitons. Again we consider both the dissipative and the inertial limits.

2 The model

First we describe the dynamics of edge excitations in the FQH samples. As was shown by Wen [7], this dynamics is governed by the following chiral Luttinger liquid Lagrangian:

$$L = -\frac{\hbar}{4\pi g}\int dx(\phi_t + v\phi_x)\phi_x \ , \tag{1}$$

where v is the velocity of the edge excitation, $g = \nu$ is the filling factor and $\phi(x)$ is a 1-D bosonic field, whose physical meaning is given by the two following relations: $\rho = e\phi_x/2\pi$ and $j = -e\phi_t/2\pi$. Here ρ is the charge density, while j is the edge current. Writing the Lagrangian this way, one assumes that the edge current direction coincides with the direction of the x-axis. The model (1) is quantized by assuming the following commutation relations:

$$[\phi(x), \phi(y)] = \pi g \,\mathrm{sgn}(x-y) \ , \tag{2}$$

$$[\pi_\phi(x), \phi(y)] = \frac{\hbar}{2}\delta(x-y) \ , \tag{3}$$

where $\pi_\phi \equiv \delta L/\delta\phi_t$. The remarkable property of the edge excitations that we will be using later is the fact that the non-zero charge density is always accompanied by the non-zero current, and vice versa.

Consider two parallel edges with opposite current directions (See Fig.(1a)). These may be two sides of the same FQH sample or the two edges of different samples. We will denote these edges by sub-letters R (right moving) and L (left moving). The R edge is described by the Lagrangian (1):

$$L_R = -\frac{\hbar}{4\pi g}\int dx(\phi_{Rt} + v\phi_{Rx})\phi_{Rx} \ , \tag{4}$$

while for the L edge the current direction is opposite to the x-axis direction. Thus, for the L edge:

$$L_L = \frac{\hbar}{4\pi g}\int dx(\phi_{Lt} - v\phi_{Lx})\phi_{Lx} \ . \tag{5}$$

The charge density and the current at the L edge are given by $\rho_L = -e\phi_{Lx}/2\pi$ and $j_L = -e\phi_{Lt}/2\pi$. Consider now the two edges as a unified dynamical system. The net charge density and current of the two edges are then:

$$\rho \equiv \rho_R + \rho_L = \frac{e(\phi_{Rx} - \phi_{Lx})}{2\pi} \ , \tag{6}$$

$$j \equiv j_R + j_L = -\frac{e(\phi_{Rt} + \phi_{Lt})}{2\pi} \ . \tag{7}$$

The Lagrangian of the combined system is $L = L_R + L_L$. Transforming L to the new variables $\phi = \phi_R - \phi_L$, $\theta = \phi_R + \phi_L$, one arrives at:

$$L = -\frac{\hbar}{8\pi g}\int dx(\phi_x\theta_t + \theta_x\phi_t + v\phi_x^2 + v\theta_x^2) \ . \tag{8}$$

Introducing two conjugate momenta $\pi_\phi = 2\delta L/\delta\phi_t$ and $\pi_\theta = 2\delta L/\delta\theta_t$ (factor 2 is to compensate for the 1/2 in (3)), one obtains the Hamiltonian in the ϕ representation:

$$\mathcal{H}_0 = \frac{\hbar v}{2}\int dx\left(\frac{\phi_x^2}{4\pi g} + 4\pi g\pi_\phi^2\right) \ . \tag{9}$$

The Hamiltonian (9) corresponds to the usual (non chiral) Luttinger liquid model [8]. The characteristic energy scale of (9) is the width of the band of excitations in the edge channels, $\hbar\omega_{\rm cut}$, which is limited by the gap in the bulk FQH state. As an upper bound to $\omega_{\rm cut}$ we may take the cyclotron frequency.

We consider now the two systems sketched in Fig.(1b,c). The first consists of a series of gates imposed on a Hall bar, while the second is a series of tunnel barriers connecting two FQH bars. In both systems, between each pair of neighboring barriers there is an island of mesoscopic size (taken to be much larger than the width of the barrier). We assume, for simplicity, that all the barriers and islands are equal and we denote by x_i the locations of the barriers.

For every barrier in the system we should add to the Hamiltonian a tunneling term, describing the tunneling of the charge carriers across the barrier (the back-scattering of the edge excitations). This term is usually taken as: $\mathcal{H}_B \propto \psi_L^\dagger(x_i)\psi_R(x_i) + h.c.$, where $\psi^\dagger$ is the creation operator for the tunneling charge carrier. This term might contain a phase shift brought about by the coupling of the charge carriers to the external magnetic field [9]. In case I the tunneling charge carriers are Laughlin quasiparticles (or vortices), thus ${\psi_R}^\dagger \propto e^{i\phi_R}$ (see [7]), and the tunneling Hamiltonian is:

$$\mathcal{H}_{B,I} = V_{B,I}\sum_i \cos\left[\phi_R(x_i) - \phi_L(x_i)\right] = V_B \sum_i \cos[\phi(x_i)] \ . \qquad (10)$$

Here $\phi_i \equiv \phi(x_i) = \phi_R(x_i) - \phi_L(x_i)$ describes the charge that was brought to the barrier at x_i. The tunneling energy scale is the height of the barrier, $V_{B,I} \sim \hbar\omega_{\rm cut}|r|^2$, where r is the reflection amplitude of the barrier. In this case there is no phase shift in the tunneling Hamiltonian (10). This may be understood using the Ginsburg-Landau-Chern-Simons (GLCS) mean field theory of the FQH effect [10] or the dual form of this theory [11]. The vortices in the GLCS theory are topological excitations of a charged boson field, which is coupled to a gauge field composed of the external magnetic field A and a statistical field a. The mean field solution implies that $< A + a >= 0$. The charged vortices thus do not feel any average gauge field, and do not acquire any phase while moving through the FQH liquid.

In case II the tunneling particles are electrons, ${\psi_R}^\dagger \propto e^{i\phi_R/g}$, and the tunneling is through potential barriers, where no strongly correlated electron liquid is present. Therefore the tunneling electrons feel only the external magnetic field and the corresponding phase factor should be taken into account. The tunneling Hamiltonian is thus:

$$\mathcal{H}_{B,II} = V_{B,II}\sum_i \cos\left[\frac{\phi(x_i)}{g} + \frac{2\pi\Phi_i}{\Phi_0}\right] \ . \qquad (11)$$

Here, $V_{B,II}$ stands for the interbar tunneling energy scale. Φ_i is the magnetic flux through all the islands (the places not filled with the FQH liquid) situated to the left of the tunnel ith barrier, and $\Phi_0 \equiv h/e$ is the flux quantum. The fluxes Φ_i are construction dependent. If the magnetic field in the islands is of the same order of magnitude as the magnetic field in the FQH strips (the most probable experimental situation) the flux through an island is very large, so the phase factor in the tunneling Hamiltonian is random (modulus 2π). One may think, however, of a different experimental setup where the fluxes Φ_i are under control. Consider, for example, two FQH strips parallel to the $x-y$ plane shifted vertically (in the z direction) from each other. If the modulations at the edges of the two strips overlap, the tunneling between the edges is in the z direction. In such a system all Φ_i's are zero. In what follows we concentrate on this (zero phase shifts) situation. The random phase shifts situation resembles the effect of offset charges in Josephson junction arrays [9], and seems to produce a completely different physics. We will consider this situation elsewhere.

Next, we should take into account the real charging energy of the system, caused by local charge fluctuations in the array. For a discrete system (like ours), we find it natural to define the charging energy per island. Thus we add to the Hamiltonian:

$$\mathcal{H}_{C_0} = \sum_i \frac{E_{C_0}}{(2\pi)^2} [\phi(x_i) - \phi(x_{i+1})]^2 , \tag{12}$$

which is the charging energy of the islands. Its energy scale is $E_{C_0} = e^2/2C_0$, where C_0 is either the capacitance of an island to a substrate or the self capacitance of the island. In the first case, this capacitance scales like l (l is the length of the island) for a given width of the system. In the second case, it scales with the linear size of the island.

So far, we considered three energy scales. An additional energy scale which plays a role is the energy level spacing of the edge states within each island, $\Delta\epsilon \sim \hbar v/l$. We show below that the role of this scale is to determine the crossover between inertial and dissipative behavior of the system.

3 Description in terms of barriers' degrees of freedom

Following the procedure used in [12], we can integrate out the degrees of freedom at positions other than the barriers. We thus obtain an effective Euclidean action in terms of the variable ϕ_i. The charging energy and the back-scattering terms do not change:

$$S_{C_0} = \sum_i \int d\tau \, \frac{E_{C_0}}{(2\pi)^2} (\phi_i - \phi_{i+1})^2 , \tag{13}$$

$$S_{B,I} = -\sum_i \int d\tau \, V_{B,I} \cos(\phi_i) . \tag{14}$$

$$S_{B,II} = -\sum_i \int d\tau \, V_{B,II} \cos\left(\phi_i/g\right) . \tag{15}$$

In the new coordinates, the self charging and the back-scattering energies may be understood as a longitudinal coupling energy between the barriers, and as a potential energy, respectively. The potential energy is in the form of a periodic charging energy resulting from non-linear capacitor with capacitance $C_{B,I} = e^2/(2\pi)^2 V_{B,I}$ in case I and $C_{B,II} = (ge)^2/(2\pi)^2 V_{B,II}$ in case II.

The integration in the free parts of the Hamiltonian gives:

$$\begin{aligned} S_0 \;\; = \;\; & \frac{k_B T}{\hbar} \sum_i \sum_n \frac{\hbar v}{2(4\pi g)} \left[\frac{\alpha_n}{\tanh(\alpha_n l)} \left(|\phi_{i_n}|^2 + |\phi_{i+1_n}|^2 \right) \right. \\ & \left. - \frac{\alpha_n}{\sinh(\alpha_n l)} 2\mathrm{Re}(\phi^*_{i_n} \phi_{i+1_n}) \right] , \end{aligned} \tag{16}$$

where k_B is Boltzman's constant, $\alpha_n \equiv \omega_n/v$, ω_n are Matsubara frequencies, and ϕ_{i_n} is a Fourier component of $\phi(\tau)$. The coefficients of expression (16) have a crossover from parabolic to linear behavior at a critical frequency $\Omega_{\rm cr} = v/l$. This critical frequency corresponds to $\Delta\epsilon$. When $\omega_1 > \Omega_{\rm cr}$, i.e., for long islands and high temperature, the action (16) becomes dissipative:

$$S_{0_{\rm diss}} = \frac{k_B T}{\hbar} \sum_i \sum_n \frac{\hbar}{4\pi g} |\omega_n| |\phi_{i_n}|^2 \ . \tag{17}$$

This dissipation is of the standard, ohmic form[13], and it is a generalization of the result obtained in [12]. If, on the other hand, several ω_n are in the parabolic section (short islands and low temperature), we get instead of dissipation a coupling term

$$S_{0_C} = \sum_i \int d\tau \, \frac{1}{2} \frac{\hbar v}{(4\pi g) l} (\phi_i - \phi_{i+1})^2 \ , \tag{18}$$

and a kinetic term

$$S_{0_{\rm kin}} = \sum_i \int d\tau \, \frac{1}{2} \frac{\hbar l}{(4\pi g) v} \frac{1}{3} (\phi_{i,t}^2 + \phi_{i+1,t}^2 + \phi_{i,t}\phi_{i+1,t}) \ . \tag{19}$$

If the length of the islands is of the order of 10^3 Å, the temperature which separating dissipative from inertial behavior is of the order of 1 K. Regarding the two terms in the inertial limit as a charging and an inductive energy terms, respectively, we can define an internal Hall (or Luttinger) capacitance, $C_H = ge^2 l/\pi\hbar v$, and an internal Hall (or Luttinger) inductance, $L_H = \pi\hbar l/gve^2$. We can express these two properties in a very simple way by using the propagation time in the grains, $\tau_P = l/v$, and the Hall (or Luttinger) resistance, $R_H = h/ge^2$: $C_H = 2\tau_P/R_H$, and $L_H = \tau_P R_H/2$. The total charging energy of the system, $\tilde{E}_{C_0} = e^2/2\tilde{C}_0$, is the sum of the internal and the islands charging energies. The total capacitance is $\tilde{C}_0 = C_H/G^2$, where $G^2 \equiv 1 + C_H/C_0$.

The above discussion suggests that, under the condition we assume here, the array can be represented by the equivalent circuit shown in Fig. (2). The non-linear capacitors represent the backwards scattering at the barriers, and the linear capacitors represent the charging energy of the islands which couples neighboring barriers. The islands are represented by inductors or by resistors, according to whether the system is inertial or dissipative, respectively.

In the limit $\tilde{E}_{C_0} \gg V_B$, the characteristic length scale over which the variable ϕ_i changes is much larger than the length of the unit cell of the array (which is, approximately, the size of an island, l). If we assume that the fluxes through the islands, Φ_i, are all zero (constant) in both cases, we can take the continuum limit by replacing $(\phi_i - \phi_{i+1})^2$ by $l^2(\partial_x\phi)^2$, and obtain a pure or an overdamped sine-Gordon model for the inertial or the dissipative

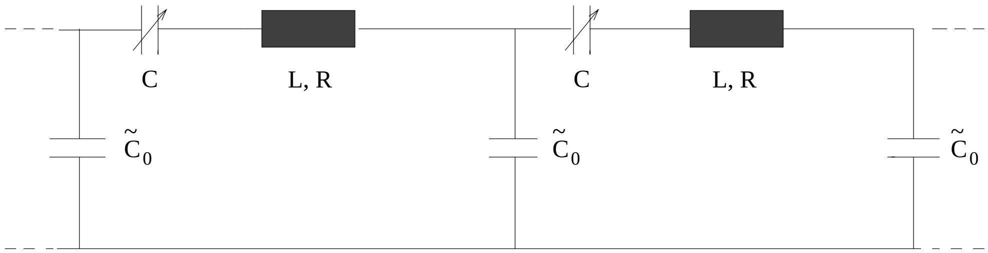

Figure 2: Equivalent circuit of the system. The black box is an inductor in the inertial regime and a resistor in the dissipative regime.

regimes, respectively. In the inertial limit, the pure sine-Gordon Lagrangian is

$$L_{I,II} = \frac{\hbar v_C}{2} \frac{1}{\beta^2_{I,II}} \int dx \left\{ \frac{1}{v_C^2} \phi^2_{t\,I,II} - \phi^2_{x\,I,II} - \frac{1}{\Lambda^2_{C,\,I,II}} [1 - \cos(\phi_{I,II})] \right\} \ , \quad (20)$$

where

$$\phi_I = g\phi_{II} = \phi \ , \quad (21)$$

$$\beta_I^2 = g^2 \beta_{II}^2 = \frac{4\pi g}{G^2} \ , \quad (22)$$

$$v_C^2 = v^2 G^2 = \frac{l}{L_H \tilde{C}_0} \ , \quad (23)$$

$$\Lambda^2_{C,\,I} = \frac{\hbar v l G^2}{4\pi g V_{B,\,I}} = l^2 \frac{C_{B,\,I}}{\tilde{C}_0} \ . \quad (24)$$

$$\Lambda^2_{C,\,II} = \frac{\hbar v l g G^2}{4\pi V_{B,\,II}} = l^2 \frac{C_{B,\,II}}{\tilde{C}_0} \ . \quad (25)$$

The meaning of the parameters β^2, v_C and Λ_C is explained below. We see that in both cases the charging energy of the islands renormalizes the parameters of the system.

4 Charge Solitons

The two sine-Gordon models presented above admit the existence of topological solitons, connecting two adjacent minima of the potential. As ϕ is a charge field, these are charge solitons. In case I, where the tunneling charge is ge, the soliton's charge is e, as follows from Eq. (6). The soliton of case I is thus an integer charge soliton. It represents the effect of an excess electron in the system, similar to charge solitons in arrays of normal [3] and Josephson [4] junctions. The number of these solitons can be controlled by introducing a gate voltage to one of the islands. However, in contrast to charge solitons in arrays of tunnel junctions, the integer charge soliton we study here involves current loops in the system, even in a static configuration. As was mentioned above, a non-zero charge density must be accompanied by a proportional current along the edge. Thus to have the inhomogeneous charge density needed for the soliton, the corresponding edge currents must be partially shortened by tunnel currents at the barriers. As a consequence, the integer charge soliton carries a magnetic flux as well. One should not confuse this flux with the additional external magnetic flux "occupied" by the FQH liquid due to the existence of the soliton. When an electron is injected into the system, the incompressible FQH liquid expands in order to maintain its charge density, and "occupies" an additional external magnetic flux of Φ_0/g.

In case II, where the tunneling charge is e, the soliton's charge is ge ((6), (21)). Therefore it is a *fractional* charge soliton. As such, it can not represent the effect of an external (integer) charge. In fact it exists in a neutral system. This fractional charge soliton is basically a Laughlin vortex located between the FQH strips, and it can be created by changing the external magnetic field. Thus the fractional charge soliton carries a magnetic flux as well, which is equal to Φ_0. This soliton can be viewed as an analogue to a fluxon in a 1D array of parallely coupled Josephson junctions.

The width of the two kind of solitons, $\Lambda_{C,I,II}$ ((24),(25)), is the characteristic length scale of the charge density (or current) modulations. The condition for assuming the continuum limit can thus be written as $\Lambda_{C,I,II} \gg l$ [4]. If this condition is not satisfied, the discreteness nature of the systems should be observed [5], [14]. Here we study only the continuum limit.

In the inertial limit the soliton is a relativistic object with a limiting velocity v_C (23) and a mass

$$M_{0,I,II} = \frac{8}{(2\pi)^2} e^2 \frac{L_H}{l} \frac{1}{\Lambda_{C,I,II}} \; . \tag{26}$$

Both v_C and $M_{0,I,II}$ depend on E_{C_0}. Typically, v_C is about two orders of magnitude less than the vacuum light velocity, and $M_{0,I,II}$ is several orders of magnitude less than the electron rest mass.

The existence of solitons in the inertial limit depends on the value of the coupling constant β^2. When $\beta^2 = 8\pi$ the ground state of the sine-Gordon model becomes unstable [15]. This is a point of a phase transition in the corresponding X-Y model [16]. In the $\beta^2 < 8\pi$ phase, isolated solitons can exist. In the $\beta^2 > 8\pi$ phase, solitons and anti-solitons are bound in dipoles. From the expression for β^2 (22), we notice that for a negligible self charging energy ($E_{C_0} = 0$) the first system is in the free solitons phase, while the second one is in the bound solitons phase (if $g < 1/2$). However, when E_{C_0} increases, the value of β^2 decreases. Thus we find that a finite self charging energy drives the system towards the free solitons phase. System I remains in its initial phase when E_{C_0} increases, but system II undergoes a phase transition: when $C_0 < C_H 2g/(1-2g)$, it is in the free solitons phase.

An exact treatment of the non linear effects, in the presence of dissipation, is not possible. We can, however, integrate out the high frequency degrees of freedom, by the methods used, separately, for the inertial sine-Gordon chain and for the dissipative single Luttinger junction. The effects of dissipation are formally the same as those induced by the presence of an infinite number of junctions. Let us integrate out the modes with frequencies $\omega_{cut} - d\omega_{cut} < \omega < \omega_{cut}$, lying in the corresponding shell of the wave numbers k. From (12) and (17) we find that we have to replace $\Lambda^2_{C,\,I,II}$ by:

$$\Lambda'^{-2}_{C,\,I,II} = \Lambda^{-2}_{C,\,I,II} e^{-K/2} \;, \tag{27}$$

where:

$$K = \int_{d\omega_{\rm cut}} \frac{dk}{2\pi}\frac{d\omega}{2\pi} \left[\frac{2E_{C_0} l k^2}{(2\pi)^2 \hbar} + \frac{|\omega|}{2\pi g l} \right]^{-1} \;. \tag{28}$$

The integral over ω should be understood as an approximation to a sum over Matsubara frequencies in the case of low temperatures. The double integral in (28) (over k and ω), combined with the extra term in the denominator, implies that the relative strength of the cosine term (that is, in units of ω_{cut}) always grows, suppressing the fluctuations in the ϕ_i's. Thus, in the presence of dissipation, solitons are always in the semiclassical regime.

The small mass of the soliton and the absence of dissipation in the underdamped regime suggests that the soliton can propagate coherently over the entire array. The soliton can be quantized semi-classically using collective coordinates, as was done in [4]. Quantum effects of solitons in this regime will be studied elsewhere.

In the overdamped sine-Gordon case (the dissipative regime) one needs to apply an external voltage in order to obtain propagation of solitons, i.e., a current. If we assume that this voltage is low enough so that the soliton is not deformed, the steady state current is proportional to the voltage with an effective resistance

$$R_{{\rm eff},\,I,II} = \frac{8}{(2\pi)^2} \frac{L^2_{\rm tot}}{\Lambda_{C,\,I,II}\, l} R_H \;, \tag{29}$$

where $L_{\rm tot}$ is the length of the array. If we assume $L_{\rm tot} \approx 10\Lambda_C$, and $l \approx 10^3$Å, the effective resistance is larger than the internal resistance by three orders of magnitude. Here, as well, a future study is needed.

5 Conclusions

We have studied the charge dynamics in arrays of junctions in systems which are in the fractional quantum Hall regime. The arrays exhibit quantized charge transport, like in arrays of low capacitance Josephson or normal tunnel junctions. The unit of charge, however, can be fractional, reflecting the nature of the excitations of the system.

The existence of quantized charge solitons makes possible the use of these devices for the same purposes as the single electron circuits (electrometers, transistors, turnstiles) widely discussed in the literature. It is interesting to note that, in the present case, cotunneling effects are sharply reduced, with respect to devices based on independent electron tunneling. The non Fermi nature of the charge carriers leads to hopping amplitudes which scale to zero at small frequencies or temperatures. Hence, coherent transport across two junctions (cotunneling) is suppressed by extra powers, with respect to the case of normal electrons.

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References

[1] *The Quantum Hall Effect*, edited by R. E. Prange and S. M. Girvin (Springer-Verlag, New York, 1990).

[2] Z. Hermon, A. Stern and E. Ben-Jacob, Phys. Rev. B **49**, 9757 (1994); Z. Hermon, A. Shnirman, E. Ben-Jacob, Phys. Rev. Lett. **74**, 4915 (1995).

[3] E. Ben-Jacob, K. Mullen and M. Amman, Phys. Lett. A **135**, 390 (1989); M. Amman, E. Ben-Jacob and K. Mullen, Phys. Lett. A **142**, 431 (1989); M. Amman, E. Ben-Jacob and Z. Hermon, in *Single Electron Tunneling and Mesoscopic Devices*, edited by H. Koch and H. Lübbig (Springer-Verlag, Berlin, 1992).

[4] Z. Hermon, E. Ben-Jacob and G. Schön, Phys. Rev. B **54**, 1234 (1996).

[5] L. P. Kouwenhoven, F. W. J. Hekking, B. J. van Wess, C. J. P. M. Harmans, C. E. Timmering and C. T. Foxon, Phys. Rev. Lett. **65**, 361 (1990).

[6] X. G. Wen, Phys. Rev. B **41**, 12838 (1990).

[7] X. G. Wen, Int. J. Mod. Phys. B **6**, 1711 (1992); X. G. Wen, Adv. Phys. **44**, 405 (1995).

[8] F. D. M. Haldane, Phys. Rev. Lett. **47**, 1840 (1981).

[9] M. P. A. Fisher, private communication.

[10] S.C. Zhang, T.H. Hansson and S. Kivelson, Phys. Rev. Lett. **62**, 82 (1989); S.C. Zhang, Int. Journ. Mod. Phys. B **6**, 25 (1992).

[11] X. G. Wen and Q. Niu, Phys.Rev. B **41**, 9377 (1990).

[12] C. L. Kane and M. P. A. Fisher, Phys. Rev. Lett. **68** 1220 (1992); C. L. Kane and M. P. A. Fisher, Phys. Rev. B **46**, 15233 (1992); F. Guinea, G. Gómez-Santos, M. Sassetti and M. Ueda, Europhys. Lett. **30**, 561 (1995).

[13] A. O. Caldeira and A. J. Leggett, Phys. Rev. Lett. **46**, 211 (1981); Ann. Phys. **149**, 374 (1983).

[14] J. F. Currie, S. E. Trullinger, A. R. Bishop and J. A. Krumhansl, Phys. Rev. B **15**, 5567 (1977); M. Peyrard and M. D. Kruskal, Physica D **14**, 88 (1984).

[15] S. Coleman, Phys. Rev. D **11**, 2088 (1975).

[16] S. Samuel, Phys. Rev. D **18**, 1916 (1978); D. J. Amit, Y. Y. Goldschmidt and G. Grinstein, J. Phys. A **13**, 585 (1980).